Statistics

Lecture 9

Feb 19-8:47 AM

Class Quiz 5

x	$P(x)$
1	.3
2	.1
3	.2
4	.1
5	.3

1) Find $P(x=3)$

$= 1 - [.3 + .1 + .1 + .3] = 1 - .8 = \boxed{.2}$

2) Find σ^2 in reduced fraction.

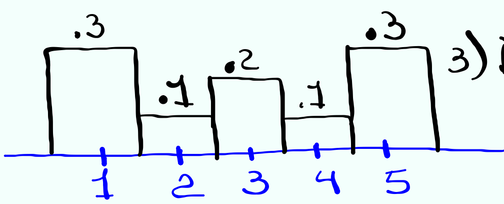
$x \to L1$, $P(x) \to L2$

$\boxed{\text{1-Var Stats}}$ with L1 & L2

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4{:}\ \sigma x}$

$\boxed{x^2}$ $\boxed{\text{Math}}$ $\boxed{1{:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$ $\frac{13}{5}$

3) Draw Prob. dist. histogram.



Oct 18-2:31 PM

Binomial Prob. Dist

n independent events

p Prob. of Success per event $q = 1 - p$

x # of Successes

$${}_{20}C_{14} = 38760$$

$$\boxed{P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}}$$

Consider a binomial Prob. dist with $n = 20$ and $p = .6$.

$$P(x=14) = {}_{20}C_{14} \cdot (.6)^{14} \cdot (.4)^{6} = \boxed{.124}$$

Using TI Command

2nd VARS

$$P(x = 14) = \text{binompdf}(\overset{n}{20}, \overset{p}{.6}, \overset{x}{14}) = \boxed{.124}$$

Oct 25-11:44 AM

I flipped a loaded Coin 50 times.

Success is to land tails

P(land tails) = .7

$n = 50$ $p = .7$ $q = .3$

$np = 50(.7) = \boxed{35}$ $npq = 50(.7)(.3) = \boxed{10.5}$ $\sqrt{npq} = \sqrt{10.5} \approx 3.24$

P(land exactly 38 tails)

$$P(x = 38) = \text{binompdf}(50, .7, 38) = \boxed{.084}$$

P(at most 40 tails)

$$P(x \le 40) = \text{binomcdf}(50, .7, 40) = \boxed{.960}$$

P(at least 30 tails)

$$P(x \ge 30) = 1 - P(x \le 29) = 1 - \text{binomcdf}(50, .7, 29) = \boxed{.952}$$

Total Prob.

We don't want 29 | 30 We want

Oct 25-11:53 AM

Consider a binomial Prob. dist. with $n=250$ and $p=.8$

1) $q=1-p=\boxed{.2}$

2) $np=250(.8)=\boxed{200}$

3) $npq=250(.8)(.2)=\boxed{40}$

4) $\sqrt{npq}=\sqrt{40}\approx\boxed{6.325}$

5) P($\underbrace{\text{fewer than 210}}$ successes)

$P(x<210)=P(x\le 209)$

$=\text{binomcdf}(250,.8,209)$

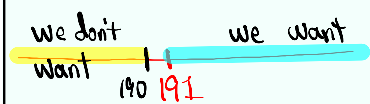
$=\boxed{.936}$

6) P($\underbrace{\text{more than 190}}$ successes)

$P(x>190)=P(x\ge 191)$

$=1-P(x\le 190)$

Total Prob.



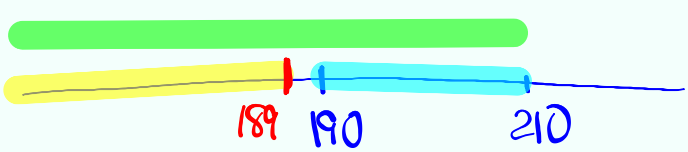
$=1-\text{binomcdf}(250,.8,190)$

$=\boxed{.931}$

Oct 25-12:04 PM

P(# of successes is between 190 and 210, inclusive)

$P(190\le x\le 210)=P(x\le 210)-P(x\le 189)$



$=\text{binomcdf}(250,.8,210)-\text{binomcdf}(250,.8,189)$

$=\boxed{.904}$

Oct 25-12:12 PM

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard deviation $\sigma = \sqrt{\sigma^2}$

Binomial Prob. Dist.

You are taking an exam of 100 True/False questions.

You are making random guesses.

Success is to guess correctly.

1) $n = 100$ 2) $P = .5$ 3) $q = .5$

4) $\mu = np$
$= 100(.5) = \boxed{50}$

5) $\sigma^2 = npq$
$= 100(.5)(.5) = \boxed{25}$

6) $\sigma = \sqrt{\sigma^2}$
$= \sqrt{25} = \boxed{5}$

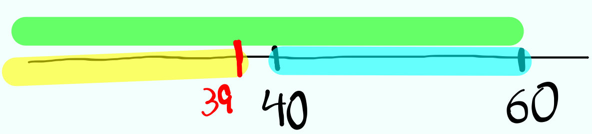
68% Range $\mu \pm \sigma = 50 \pm 5 \Rightarrow \boxed{45 \text{ to } 55}$

Usual Range
95% Range $\mu \pm 2\sigma = 50 \pm 2(5) \Rightarrow \boxed{40 \text{ to } 60}$

Oct 25-12:17 PM

P(guess between 40 and 60, inclusive, correct answers)

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$



$= \text{binomcdf}(100, .5, 60) - \text{binomcdf}(100, .5, 39)$

$= \boxed{.965} \approx 96.5\%$

Oct 25-12:25 PM

Consider a binomial Prob. dist. with
$n = 175$ & $P = .6$

1) $q = .4$ 2) $\mu = np (175)(.6) = \boxed{105}$

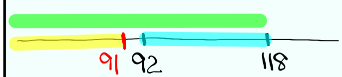
3) $\sigma^2 = npq = 175(.6)(.4)$
$= \boxed{42}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{42}$
$\approx \boxed{6.5}$

5) Find $\overbrace{\text{usual Range}}^{95\%\ \text{Range}}$

$\mu \pm 2\sigma$
$= 105 \pm 2(6.5)$ ✓
$= 105 \pm 13 \Rightarrow \boxed{92 \text{ to } 118}$

6) $P(\overbrace{\#\ \text{of Successes}}^{x}$ is between 92 and 118, inclusive$) = P(92 \le x \le 118)$



$= \text{binomcdf}(175, .6, 118) -$ $\approx 96\%$
$\text{binomcdf}(175, .6, 91) = \boxed{.963}$

SG 16 ✓

on Page 4, make Sure to use exact Value for P & q.

Oct 25-12:29 PM

SG 17

Geometric Prob. dist.

It is very Similar to binomial prob. dist.

except
1) There is no n.

2) x is the event where first Success happens.

$P \rightarrow$ Prob. of Success $P + q = 1$

$q \rightarrow$ Prob. of failure $q = 1 - P$

$$\boxed{P(x) = P \cdot q^{x-1}} \quad x = 1, 2, 3, 4, \ldots$$

$\mu = \frac{1}{P}$ $\sigma^2 = \frac{q}{P^2}$ $\sigma = \sqrt{\sigma^2}$

Oct 25-12:51 PM

Consider a geometric Prob. dist. with $p = .5$

$q = 1 - p = .5$ $\quad \mu = \frac{1}{p} = \frac{1}{.5} = 2$

$\sigma^2 = \frac{q}{p^2} = \frac{.5}{.5^2} = 2$ $\quad \sigma = \sqrt{\sigma^2} = \sqrt{2} \approx 1.414$
$\approx \boxed{1}$

68% Range $\quad \mu \pm \sigma = 2 \pm 1 \Rightarrow \boxed{1 \text{ to } 3}$

P(first success happens on 3rd trial)

$P(x = 3) = (.5)(.5)^{3-1} = .5(.5)^2 = \boxed{.125}$

geomet pdf$(.5, 3) = \boxed{}$

P(first success happens before 3rd trial)

$P(x < 3) = P(x \leq 2)$

geometcdf$(.5, 2) = \boxed{.75}$

Oct 25-12:56 PM

P(landing tails) = .6 on a loaded Coin

$p = .6$ $\quad q = .4$

$\mu = \frac{1}{p} = \frac{1}{.6} = 1.\bar{6} \approx 2$

$\sigma^2 = \frac{q}{p^2} = \frac{.4}{.6^2} = 1.\bar{1}$

$\sigma = \sqrt{\sigma^2} = \sqrt{1.111} \approx 1$

Usual Range

$\mu \pm 2\sigma$

$= 2 \pm 2(1)$

$\Rightarrow \boxed{0 \text{ to } 4}$

P(it lands tails after the 3rd trial)

$P(x > 3) = P(x \geq 4) = 1 - P(x \leq 3)$

Total Prob.

we don't want $\quad$ we want

3 4

$= 1 -$ geometcdf$(.6, 3)$

$= \boxed{.064}$

Oct 25-1:04 PM

Prob. that LeBron James makes his FT is .8.

P(He makes his first FT on 2nd attempt)

$P(x=2) = \text{geometpdf}(.8, 2) = \boxed{.16}$

P(He makes his first FT before the 5th attempt) =

$P(x<5) = P(x \le 4)$

$= \text{geometcdf}(.8, 4) = \boxed{.998}$

Oct 25-1:11 PM

Poisson Prob. dist μ, λ

Use this when average # of Successes over a fixed interval is given.

x is # of Successes

$x = 0, 1, 2, 3, \ldots$

$$\boxed{P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}}$$

$e \approx 2.718$

$\sigma^2 = \mu$ $\sigma = \sqrt{\sigma^2}$

Oct 25-1:16 PM

Consider a poisson Prob. dist with $\mu = 9$ over a fixed interval.

$\sigma^2 = \mu = 9$ $\qquad$ $\sigma = \sqrt{\sigma^2} = \sqrt{9} = 3$

95% Range $\qquad$ $\mu \pm 2\sigma = 9 \pm 2(3)$

$= 9 \pm 6$

$= \boxed{3 \text{ to } 15}$

P(exactly 10 Successes on that fixed interval)

$P(x = 10) = \text{Poisson Pdf}(9, 10)$

$= \boxed{.119}$

P(at most 12 Successes on that fixed interval)

$P(x \leq 12) = \text{Poisson Cdf}(9, 12) = \boxed{.876}$

Oct 25-1:20 PM

How many orders $\underbrace{\text{Per hour}}_{\text{fixed interval}}$ in $\underbrace{\text{average}}_{\mu=16}$

$\mu = 16$

$\sigma^2 = \mu = 16$ $\qquad$ $\sigma = \sqrt{\sigma^2} = \sqrt{16} = \boxed{4}$

68% Range $\quad$ $\mu \pm \sigma = 16 \pm 4 \Rightarrow \boxed{12 \text{ to } 20}$

P(at most 20 orders per hour)

$P(x \leq 20) = \text{Poisson Cdf}(16, 20)$

$= \boxed{.868}$

SG 17 ✓

Oct 25-1:26 PM

SG 18

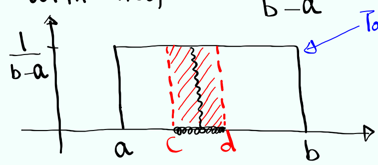
Continuous Random Variable with Prob. dist.

Uniform Prob. dist. for all values from a to b.

$a \le x < b$

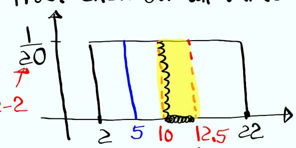
Graph is rectangular from a to b with height of $\frac{1}{b-a}$.

Total Area = 1



$P(x=c)=0$ $\qquad P(c<x<d)=(d-c)\cdot\frac{1}{b-a}$

Consider a uniform Prob. dist. for all values from 2 to 22.



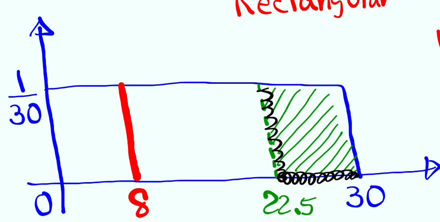
1) $P(x=5)=0$

line (zero Area)

2) $P(10<x<12.5)=(12.5-10)\cdot\frac{1}{20}=\frac{2.5}{20}=\boxed{\frac{1}{8}}$

Oct 25-1:47 PM

Consider a $\underbrace{\text{Uniform Prob. dist.}}_{\text{Rectangular}}$ for $0 \le x \le 30$



1) $P(x=8)=0$

2) $P(x>22.5)$

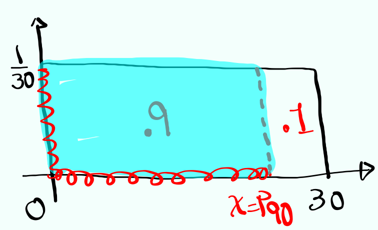
$=(30-22.5)\cdot\frac{1}{30}$

$=\frac{7.5}{30}=\boxed{\frac{1}{4}}$

3) $x = P_{90}$

90% below — Left Area .9

10% above — Right Area .1



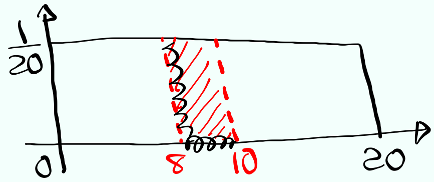
$(x-0)\cdot\frac{1}{30}=.9$

$x=30(.9) \rightarrow \boxed{x=27}$

Oct 25-1:56 PM

City bus comes around every 20 minutes.

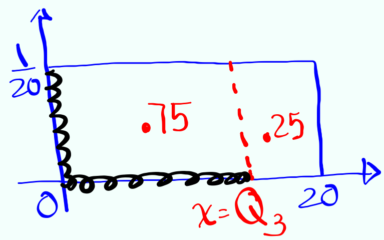
Let x be the wait time. $0 \le x \le 20$



1) $P(8 < x < 10) = (10-8)\cdot\frac{1}{20} = \frac{2}{20} = \frac{1}{10} = .1$

2) $x = Q_3$

75% below   25% above



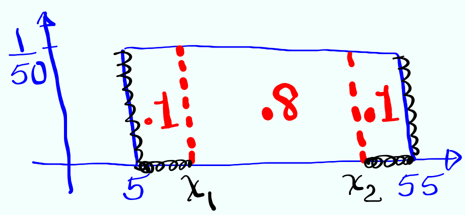
$(x-0)\cdot\frac{1}{20} = .75$

$x = 20(.75) = \boxed{15}$

Oct 25-2:03 PM

Consider a uniform Prob. dist. for all values from 5 to 55.

Find two values that separate the $\boxed{\text{middle } 80\%}$ from the rest.



10

$(x_1 - 5)\cdot\frac{1}{50} = .1$

$x_1 - 5 = 50(.1)$

$x_1 = 5 + 5$

$\boxed{x_1 = 10}$

$(55 - x_2)\cdot\frac{1}{50} = .1$

$55 - x_2 = 50(.1)$

$55 - x_2 = 5$

$\boxed{x_2 = 50}$

Oct 25-2:09 PM

Standard Normal Prob. Dist.

1) Use Z, $P(Z=c)=0$

2) Data dist is symmetric with bell-shape curve with total Area $=1$

3) Mean $=$ Mode $=$ Median

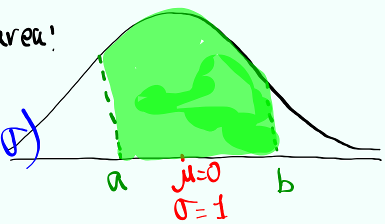
4) $\mu=0$, $\sigma=1$

$P(a<Z<b)$ is the Corresponding area within the bell-Curve.

How to find the area:

2nd VARS

normalcdf(L, U, μ, σ)

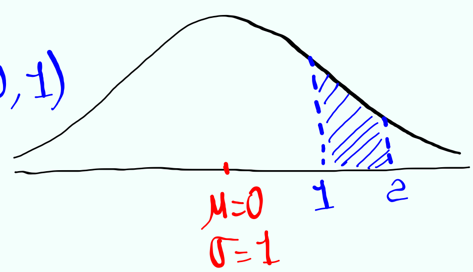


Oct 25-2:14 PM

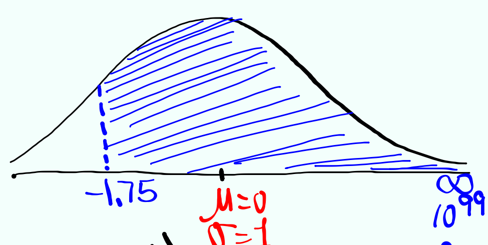
$P(1<Z<2)$

$=$ normalcdf$(1, 2, 0, 1)$

$=\boxed{.136}$



$P(Z>-1.75)$



$=$ normalcdf$(-1.75, E99, 0, 1)$

$(-)$

$=\boxed{.960}$

E99

$\boxed{2nd}\boxed{,}$

Oct 25-2:21 PM

$P(z < 1.645)$

$= \text{normalcdf}(-E99, 1.645, 0, 1)$

$(-)$ $\boxed{2nd}\boxed{9}$

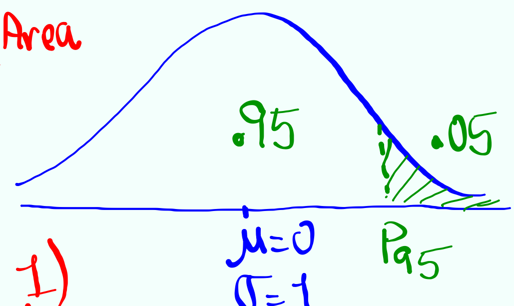
$= \boxed{.950}$

Oct 25-2:28 PM

Find $z = P_{95}$

95% below — Left Area .95

5% above — Right Area .05



Reverse Process

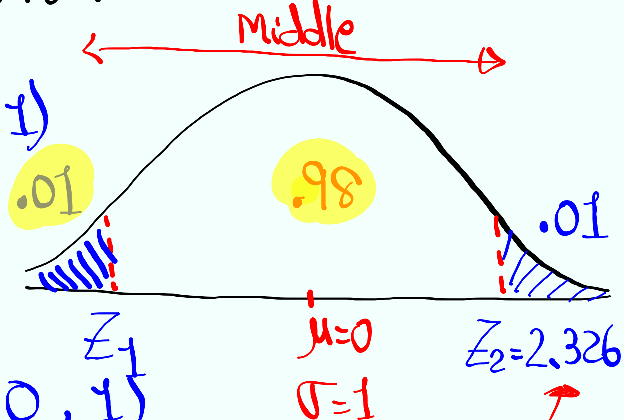
$z = \text{invNorm}(.95, 0, 1)$

left Area μ σ

$\approx \boxed{1.645}$

Oct 25-2:31 PM

Find two z-values that separate the middle 98% from the rest.



$z_1 = \text{invNorm}(.01, 0, 1)$

$= \boxed{-2.326}$

$z_2 = \text{invNorm}(.99, 0, 1)$

$= \boxed{2.326}$

Oct 25-2:35 PM